HACETTEPE UNIVERSITY

Faculty of Engineering, Department of Electrical and Electronics Engineering

MAT124 — Sect. 02 — Mathematics II

Final Examination

Student Name-Surname:	
Student ID:	
Signature:	

Date:	8 June 2026
Time:	10:00
Duration:	105 Minutes

Rules and Instructions:

1. Mobile phones and other electronic devices must be either turned off or put away.
2. Show your work clearly for the classical questions.
3. For the multiple-choice section, only one answer is correct.
4. No extra paper is allowed; use the space provided in this booklet.

Part	Q1	Q2	Q3	MC (1-8)	**Total**
Points	(15 pts)	(20 pts)	(15 pts)	(50 pts)	**100**
Score					

Part II: Classical Questions

Question 1. Use the method of Lagrange multipliers to find the absolute maximum and absolute minimum values of the function $f(x, y, z) = x^2 + 2y - z^2$ subject to the constraint $2x^2 + y^2 + z^2 = 4$. **[15 pts]**

Question 2. Consider the vector field

$$\mathbf{F}(x, y, z) = (2xe^y + z\cos(xz))\mathbf{i} + (x^2e^y - \sin(y))\mathbf{j} + (x\cos(xz) + 2z)\mathbf{k}$$

1. Show that $\mathbf{F}$ is a conservative vector field by verifying the cross-partial derivative conditions. **[5 pts]**
2. Find a potential function $\phi(x, y, z)$ such that $\mathbf{F} = \nabla\phi$. **[7,5 pts]**
3. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the path given by $\mathbf{r}(t) = \left\langle t, t(1-t), \frac{\pi}{2}t^3 \right\rangle$ for $0 \leq t \leq 1$. **[7,5 pts]**

Question 3. Let C be the upper half of the circle $x^2 + y^2 = 9$, oriented from $(3, 0)$ to $(-3, 0)$. Evaluate the line integral:

$$\int_C \left(y^2 + \sin(x^3)\right) dx + \left(3xy + e^{y^2}\right) dy$$

Hint: The curve C is not closed. Consider adding a line segment to form a closed loop and use Green's Theorem.
[15 pts]

Question 4. Answer the following questions. For multiple choice questions, mark the correct option. For others, write your answer in the box provided. No partial credits will be given. **[Credits: 10 + 6 x 5 + 10]**

4.1 Consider the iterated double integral $\int_{-8}^{8} \int_{x^{2/3}}^{4} f(x, y)\, dy\, dx$. Reverse the order of integration and write the new limits in the boxes below:

$$\int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} f(x, y)\, dy\, dx$$

4.2 Let E be the solid region bounded above by the paraboloid $z = x^2 + y^2$, below by the xy-plane ($z = 0$), and enclosed by the elliptic cylinder $x^2 + 4y^2 = 4$. What is the volume of the region E?
(Hint: Use a modified coordinate system such as $x = 2r\cos\theta$ and $y = r\sin\theta$ to simplify the region).
(A) $\frac{3\pi}{2}$ (B) 2π (C) $\frac{5\pi}{2}$ (D) 3π (E) 4π

4.3 Consider the vector field $\mathbf{F}(x, y) = \left\langle y^2, -\frac{2}{x-2} \right\rangle$ representing the velocity field of a steady fluid flow. Imagine placing a tiny paddle wheel into this fluid. The paddle wheel will rotate counter-clockwise if the scalar curl of the field is positive, and clockwise if it is negative. In which of the following regions above the x-axis ($y > 0$) will the paddle wheel definitely rotate in a **counter-clockwise** direction?

(A) $0 < y < \frac{1}{(x-2)^2}$ (B) $y^2 > \frac{2}{x-2}$ (C) $y^2 < \frac{2}{x-2}$ (D) $y > \frac{1}{(x-2)^2}$ (E) Everywhere above the x-axis.

4.4 Let D be the region in the xy-plane bounded by the lines $y = x$, $y = x - 2$, $y = -x$, and $y = -x + 3$. Using an appropriate change of variables, evaluate the double integral:

$$\iint_D (x + y)e^{x^2 - y^2}\, dA$$

(A) $\frac{1}{2}(e^6 - 1)$ (B) $\frac{1}{4}(e^6 - 7)$ (C) $e^6 - 7$ (D) $\frac{3}{2}(e^4 - 1)$ (E) $2(e^6 - 3)$

4.5 Which of the following curves represents a field line (integral curve) for the vector field $\mathbf{F}(x, y) = y\mathbf{i} + x\mathbf{j}$? (where C is a constant)

(A) $x^2 + y^2 = C$ (B) $y = Cx$ (C) $y = Ce^x$ (D) $x^2 - y^2 = C$ (E) $xy = C$

4.6 Calculate the outward flux of the vector field $\mathbf{F}(x, y, z) = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ across the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

(A) $\frac{4\pi}{5}$ (B) $\frac{8\pi}{5}$ (C) $\frac{12\pi}{5}$ (D) 4π (E) 12π

4.7 Consider the following two vector fields in $\mathbb{R}^3$:

$$\mathbf{F}_1 = \langle -y, x, 0\rangle \quad \text{and} \quad \mathbf{F}_2 = \langle x, y, z\rangle$$

How can these fields be classified?

(A) Both are solenoidal.
(B) Both are irrotational.
(C) $\mathbf{F}_1$ is solenoidal, $\mathbf{F}_2$ is irrotational.
(D) $\mathbf{F}_1$ is irrotational, $\mathbf{F}_2$ is solenoidal.
(E) Neither field is solenoidal nor irrotational.

4.8 The vector field $\mathbf{F}(x, y, z) = 2y\mathbf{i} + z\mathbf{j} + 0\mathbf{k}$ is solenoidal. Find a vector potential $\mathbf{A}$ such that $\nabla \times \mathbf{A} = \mathbf{F}$. *Assume the x-component of the vector potential is zero.* Write the remaining components in the boxes below:

$$\mathbf{A}(x, y, z) = \left\langle 0\,, \boxed{}, \boxed{} \right\rangle$$